Fully and partially non-neutral plasma equilibria in a variable-electrode-radius Malmberg-Penning trap

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Two types of plasma equilibria are self-consistently computed for a three-electrode Malmberg-Penning trap that has an increase in the radius of a section of the center electrode. When a single species, fully non-neutral plasma is confined within the trap, equilibria are predicted in which the plasma produces a three-dimensional electric potential well. Partially non-neutral plasma equilibria are predicted to be possible by confining a second, oppositely signed plasma species within the three-dimensional well produced by the first plasma species. Conditions that are necessary for partially non-neutral plasma equilibria to be self-consistently possible are reported. A partially non-neutral plasma formed of electrons and singly charged xenon ions is then specifically considered, first with the ions confined within a three-dimensional well produced by the electrons and next with the electrons confined within a three-dimensional well produced by the ions.

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A Malmberg-Penning trap is a plasma confinement apparatus that produces an electric field using at least three cylindrical electrodes, which are placed end to end and aligned with a uniform magnetic field [1]. In a common mode of operation, a one-dimensional electric potential well is generated by the electrodes. The electric potential well can provide axial confinement of a fully non-neutral plasma, which is a plasma comprised of particles having the same charge sign. Radial confinement in the other two dimensions is provided by the magnetic field. There also exists a "nested" mode of operation in which nested electric potential wells are applied using a sequence of at least five electrodes. The nested mode of operation can provide simultaneous axial confinement of oppositely signed plasma species, and the interaction between those species can be studied. Various plasma equilibria for nested Malmberg-Penning traps have been theoretically predicted [2,3], and experimental studies using nested Malmberg-Penning traps have been reported involving protons and electrons [4], and positrons and antiprotons [5–8].

A theoretical study is presented here that is divided into two parts. The first part consists of an investigation of the effect that a change in radius of the center electrode of a three-electrode Malmberg-Penning trap would have on the equilibrium of a fully non-neutral plasma. Although an electron plasma is referred to, the analysis also applies to a plasma of singly charged positive particles if the charge and electrode-voltage signs are changed. The second part of the study consists of an exploration regarding the possibility of confining a partially non-neutral plasma as a result of simultaneously trapping two oppositely signed plasma species within a variable-electrode-radius Malmberg-Penning trap. The electrode configuration considered is illustrated in Fig. 1. A cylindrical coordinate system is defined such that the z axis coincides with the axis of symmetry of the configuration. Because the configuration is azimuthally symmetric about the z axis, only the radial and axial coordinates (r,z) need to be referred to. The configuration is also symmetric about the $z=0$ midplane. The center electrode has an applied electric potential denoted by V_0, a radius r_{w0} for $|z|<z_1$, and a radius r_{w2} for $z_1<|z|<z_3$. Each of the two end electrodes, which extend axially for $z_3<|z|<z_w$, has an applied potential V_4 and a radius r_{w2}. A uniform magnetic field is present, which is aligned with the z axis. For confining an electron plasma, the end electrodes would be biased negative with respect to the center electrode $V_4<V_0$.

Before considering the effect that a change in radius of the center electrode in Fig. 1 would have on a trapped electron plasma, it is helpful to consider a cylindrical electron plasma of radius r_p that is confined within a cylindrical electrode of inner radius r_w. Assuming a uniform plasma density n and a plasma length that is much larger than the inner wall radius r_w, Gauss' law provides a suitable approximation for the radial electric field strength at any axial position that is far from the axial edges of the plasma. Gauss' law gives $E_r=-enr/(2\epsilon_0)$ for $r\leqslant r_p$ and $E_r=-enr_p^2/(2\epsilon_0 r)$ for $r_p\leqslant r<r_w$, where E_r is the radial component of the electric field, e is the unit charge, and ϵ_0 is the permittivity of free space. The difference in the electric potential between the center of the plasma at $r=0$ and the electrode inner surface at $r=r_w$ is obtained by integrating the radial electric field. Defining the electric potential to be zero at the electrode (at $r=r_w$), the potential at $r=0$ is $V_{(r=0)}=-enr_p^2[1+2\ln(r_w/r_p)]/(4\epsilon_0)$. Note the dependence that $V_{(r=0)}$ has on the inner wall radius r_w.

Now consider a cylindrical electron plasma of radius r_p trapped within the electrode configuration shown in Fig. 1,

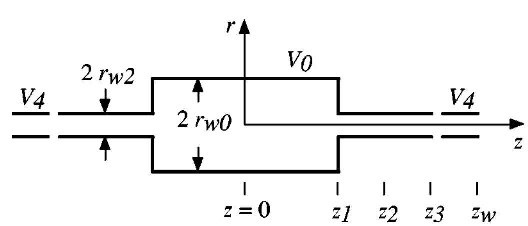

FIG. 1. A cylindrically symmetric electrode configuration having a variable-radius center electrode. Also shown is the orientation of a cylindrical coordinate system. A uniform magnetic field is present that is aligned with the z axis.

and assume $z_1 \gg r_{w0}$ and $z_3 - z_1 \gg r_{w2}$. According to the functional dependence of $V_{(r=0)}$, if the electron plasma has a uniform density within a cylindrical volume, which is of radius r_p and approximate length $2z_3$, then a minimum would exist in the electric potential at $(r,z)=(0,0)$. However, the plasma density distributes itself self-consistently, which results in Debye shielding of macroscopic electric field components that are parallel to the magnetic field. In the limit that the ratio of the plasma diameter to the plasma Debye length is infinite, no axial minimum would be expected to occur. However, if the plasma diameter is only a few times larger than the plasma Debye length, then the plasma response must be taken into account self-consistently in order to predict whether an electric potential minimum would occur.

The Boltzmann density relation describes how a relaxed plasma distributes itself in the presence of a macroscopic electric field. The Boltzmann density relation is briefly derived here in order to indicate a basic assumption associated with its use. Suppose an unmagnetized, collisionless, single-species, steady-state plasma is free of particle sources and sinks. The phase-space distribution function $f(\boldsymbol{r},\boldsymbol{v})$ for the plasma must satisfy the time-independent Vlasov equation $m\boldsymbol{v}\cdot\boldsymbol{\nabla} f(\boldsymbol{r},\boldsymbol{v}) = q\boldsymbol{\nabla}\phi(\boldsymbol{r})\cdot\boldsymbol{\nabla}_v f(\boldsymbol{r},\boldsymbol{v})$, where m and q are the mass and charge of a plasma particle, $\phi(\boldsymbol{r})$ is the electric potential at position $\boldsymbol{r}$, and $\boldsymbol{\nabla}_v$ denotes gradient in velocity space. Assume the plasma's velocity distribution function at a location denoted by $\boldsymbol{r}_s$ is Maxwellian. A Maxwellian velocity distribution function at $\boldsymbol{r}_s$ is written as $f_s(\boldsymbol{v}) = n_s[m/(2\pi T)]^{3/2}\exp[-mv^2/(2T)]$, where $n_s = n(\boldsymbol{r}_s)$ is the plasma density at $\boldsymbol{r}_s$, and T is the plasma temperature. (Plasma temperature is in energy units throughout.) The time-independent Vlasov equation is then satisfied by the Maxwell-Boltzmann phase-space distribution $f(\boldsymbol{r},\boldsymbol{v}) = n_s[m/(2\pi T)]^{3/2}\exp[-mv^2/(2T)]\exp(-q[\phi(\boldsymbol{r})-\phi(\boldsymbol{r}_s)]/T)$, which is written such that it reduces at $\boldsymbol{r}_s$ to the Maxwellian velocity distribution: $f(\boldsymbol{r}_s,\boldsymbol{v}) = f_s(\boldsymbol{v})$. Integrating the Maxwell-Boltzmann phase-space distribution over velocity space provides $n(\boldsymbol{r}) = n_s\exp(-q[\phi(\boldsymbol{r}) - \phi(\boldsymbol{r}_s)]/T)$, which is the Boltzmann density relation.

The Boltzmann density relation is now applied to a cylindrical electron plasma of radius r_p trapped within the electrode configuration shown in Fig. 1. The symbol r will continue to represent the radial coordinate of the cylindrical coordinate system. (Hence, r does not represent the magnitude of the position vector $\boldsymbol{r}$.) To take into account the presence of a uniform magnetic field, the electron plasma is treated in the guiding-center approximation with the assumption that the electron cyclotron radius is much smaller than the radial length scales associated with electric potential and electron density gradients. When applied in the axial dimension at a guiding-center radial coordinate r, the Boltzmann density relation can be written as

$$n_-(r,z) = n_{s-}h_-(r)e^{e[\phi(r,z)-\phi(r,z_2)]/T_-}, \tag{1}$$

by assuming that the electron velocity distribution is Maxwellian at $z=z_2$ at each r. Here, $n_-(r,z)$ denotes electron density at (r,z), the product $n_{s-}h_-(r)$ equals the electron density at (r,z_2), n_{s-} equals the electron density at $(0,z_2)$, $h_-(r)$ is a function that specifies the radial profile of the electron plasma at z_2 with $h_-(0)=1$, and T_- is the electron temperature, which is assumed to be radially uniform. A plasma that follows the Boltzmann density relation axially along each field line of a uniform magnetic field [e.g., Eq. (1)] has been referred to as being in ''local thermal equilibrium'' [9,10].

To self-consistently determine the electric potential within the electrode configuration in Fig. 1, Poisson's equation $\nabla^2\phi = f$ must be solved, where $f(r,z) = (e/\epsilon_0)n_-(r,z)$. A finite-differences computational approach, which has been used to predict plasma equilibria in nested well and single-well Malmberg-Penning traps [3,11], is used to numerically solve Poisson's equation with the electron density given by Eq. (1). The choice made for the radial electron density profile at z_2 is one similar to profiles commonly observed for relaxed plasmas in Malmberg-Penning traps. A profile of the form $h_-(r) = [1-(r/r_p)^\alpha]\Theta(r_p - r)$ is used with $\alpha = -2.3/\ln(1-\lambda_{Ds-}/r_p)$, which causes the electron density to decrease near the plasma edge primarily within one Debye length. Here, α is a coefficient that determines how broad (large α) or narrow (small α) the profile is, $\lambda_{Ds-} = [\epsilon_0 T_-/(e^2 n_{s-})]^{1/2}$ is the electron Debye length at $(0,z_2)$, and Θ is the Heaviside step function. In order to define a closed computation region, the separation between the center electrode and each end electrode is set equal to the grid spacing, and a vertical electrode wall is considered that caps each end electrode at $|z| = z_w$ and that has an applied potential V_4. The boundary conditions used are a zero radial electric field component at the z axis due to azimuthal symmetry, a zero axial electric field component at $z=0$ due to symmetry about the midplane, and the chosen potentials along the electrode walls. By way of example, the following parameter values are selected: $n_{s-} = 8.4\times10^{12}$ m^{-3}, $T_- = 0.5$ eV, $r_p = 0.5$ cm, $r_{w0} = 2$ cm, $r_{w2} = 0.5$ cm, $z_1 = 4$ cm, $z_2 = 6$ cm, $z_3 = 8$ cm, $z_w = 10$ cm, $V_0 = 0$ V, and $V_4 = -15$ V. The resulting numerical solution indicates that a local minimum in the electric potential does indeed occur at $(0,0)$, and an associated three-dimensional electric potential well is self-consistently produced. In Fig. 2, plots are shown of the normalized potential $\psi_- = e\phi/T_-$. The formation of axial and radial electric potential wells are apparent. Also note that two maxima occur in the plot of $\psi_-(0,z)$. The maxima occur at approximately $z = \pm z_2 = \pm 0.6 z_w$ and correspond to potential energy minima for electrons along the z axis. The potential energy minima are at the bottom of an axial potential energy well that provides axial electron confinement. The top of the axial potential energy well is located at $(z = \pm z_w)$, and the depth of the axial potential energy well can be controlled by the value chosen for V_4. The depth of the axial potential energy well at each radial coordinate r is more than an order of magnitude larger than the electron temperature. With an axial potential energy well that is much deeper than the electron temperature, electron loss regions in velocity space are negligibly small at the bottom of the well, which is consistent with the assumption that the electrons have an essentially full Maxwellian velocity distribution at $z=z_2$ at each radial coordinate.

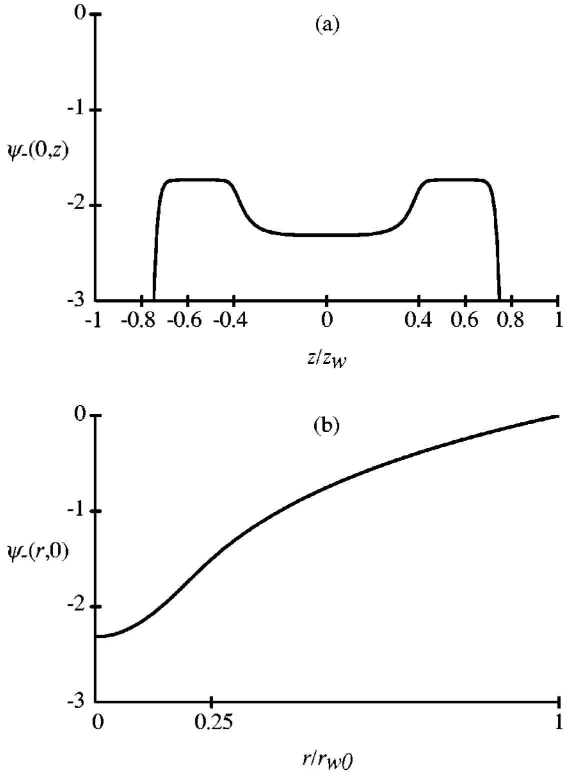

FIG. 2. Normalized electric potential along the z axis (a) and along the midplane (b). Note that $z_1/z_w = 0.4$, $z_2/z_w = 0.6$, $z_3/z_w = 0.8$, and $r_p/r_{w0} = r_{w2}/r_{w0} = 0.25$. Also, the normalized potential of the end electrodes is given by $eV_4/T_- = -30$ and is off the scale.

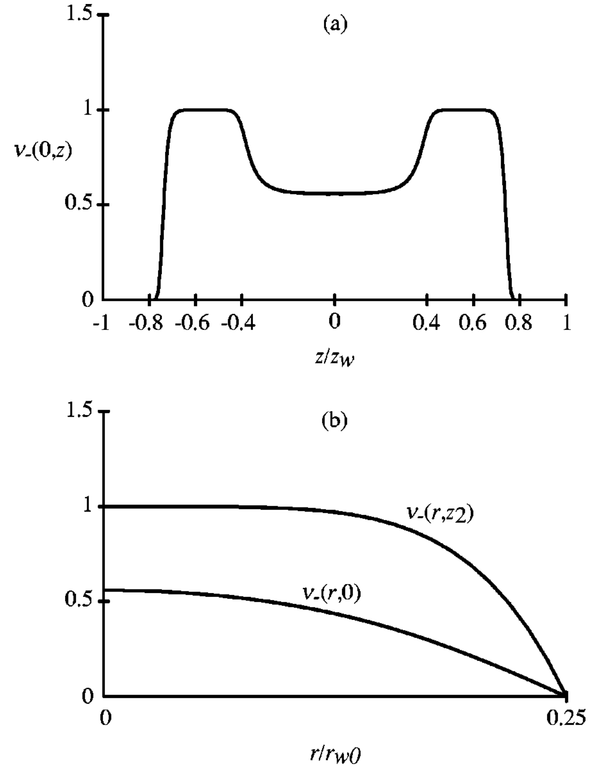

FIG. 3. Normalized electron density along the z axis (a) and at two axial coordinates (b). Note that for (b), the scale of the radial axis is different from that in Fig. 2(b), with the radial axis only extending to $r/r_{w0} = r_p/r_{w0} = r_{w2}/r_{w0} = 0.25$.

Plots are shown of the normalized electron density $\nu_-(r,z) = n_-(r,z)/n_{s-}$ in Fig. 3. Note that the electron density changes substantially near $z = \pm z_1 = \pm 0.4 z_w$, where the change in electrode radius occurs. The electron density at (0,0) is $4.7\times 10^{12}\ \mathrm{m}^{-3}$, which is about half that at $(0,z_2)$. The ratio of the plasma diameter to the electron Debye length is 5.5 if the Debye length calculated at $(0,z_2)$ and 4.1 if the Debye length calculated at (0,0). It can be concluded that a macroscopic electric field component that is parallel to the magnetic field can be self-consistently produced by the electron plasma as a result of a change in electrode radius, at least if the diameter of the electron plasma is only a few times larger than the Debye length associated with the electron plasma.

To obtain a broader applicability for the results shown in Figs. 2 and 3, Eq. (1) is substituted into Poisson's equation, and an expression in terms of dimensionless quantities is derived. The expression is

$$\nabla^2 \psi_-(R,Z) = h_-(R) e^{\psi_-(R,Z) - \psi_-(R,Z_2)}, \tag{2}$$

where the normalized potential ψ_- is written in terms of the normalized coordinates $R = r/\lambda_{Ds-}$ and $Z = z/\lambda_{Ds-}$, and the Laplacian is with respect to R and Z. Also, $h_-(R) = [1 - (R/R_p)^\alpha]\Theta(R_p - R)$, $\alpha = -2.3/\ln(1 - R_p^{-1})$, $R_p = r_p/\lambda_{Ds-}$, and $Z_2 = z_2/\lambda_{Ds-}$. The boundary conditions for the problem can be specified in terms of the following dimensionless parameters: $R_{w0} = r_{w0}/\lambda_{Ds-}$, $R_{w2} = r_{w2}/\lambda_{Ds-}$, $Z_1 = z_1/\lambda_{Ds-}$, $Z_3 = z_3/\lambda_{Ds-}$, $Z_w = z_w/\lambda_{Ds-}$, $\psi_-(R_{w0},0) = eV_0/T_-$, and $\psi_-(R_{w2},Z_w) = eV_4/T_-$. Consequently, the problem can be expressed entirely in terms of the dimensionless quantities ψ_-, R, Z, R_p, R_{w0}, R_{w2}, Z_1, Z_2, Z_3, Z_w, $\psi_-(R_{w0},0)$, and $\psi_-(R_{w2},Z_w)$, and a solution in terms of these dimensionless quantities applies for any set of chosen parameter values provided the dimensionless parameters R_p, R_{w0}, R_{w2}, Z_1, Z_2, Z_3, Z_w, $\psi_-(R_{w0},0)$, and $\psi_-(R_{w2},Z_w)$ remain unchanged. Note that the results in Figs. 2 and 3 are in terms of ψ_-, $z/z_w = Z/Z_w$, $r/r_{w0} = R/R_{w0}$, and, by Eq. (1), $\nu_- = h_-(R) e^{\psi_-(R,Z) - \psi_-(R,Z_2)}$. Thus, the results in Figs. 2 and 3 also apply, for example, for the following parameter values: $n_{s-} = 4.2\times 10^{12}\ \mathrm{m}^{-3}$, $T_- = 1$ eV, $r_p = 1$ cm, $r_{w0} = 4$ cm, $r_{w2} = 1$ cm, $z_1 = 8$ cm, $z_2 = 12$ cm, $z_3 = 16$ cm, $z_w = 20$ cm, $V_0 = 0$ V, and $V_4 = -30$ V.

A parameter study was carried out to determine the effect that changing certain parameters would have on the electric potential well shown in Fig. 2. In each case, the axial well depth remained smaller than the radial well depth, and only the effect on the normalized axial well depth $\Delta\psi_{m-} = \psi_-(r = 0, z \approx z_2) - \psi_-(r = 0, z = 0)$ is reported. Here, $\psi_-(r = 0, z \approx z_2)$ is the maximum normalized potential along the z axis. To carry out the parameter study, a number of computations were done that were the same as the computation that produced the results shown in Figs. 2 and 3 (hereafter referred to as the "single-species base case"), except with a change in one or more of the adjustable parameters. For the single-species base case, the value of $\Delta\psi_{m-}$ is 0.58. When the value for α was varied from its base case value of 5.1 without changing R_p, $\Delta\psi_{m-}$ was found to increase monotonically with α. In the limit $\alpha\rightarrow\infty$, which corresponds to a step function radial density profile $h_{-}(r)\rightarrow\Theta(r_p-r)$, $\Delta\psi_{m-}$ increased by 12% relative to its base case value. For a parabolic profile ($\alpha=2$), the value of $\Delta\psi_{m-}$ decreased by 8.9% relative to its base case value. The values of R_p, R_{w0}, R_{w2}, Z_1, Z_2, Z_3, and Z_w, were simultaneously changed in a self-similar fashion by changing n_{s-}. $\Delta\psi_{m-}$ had a maximum value when n_{s-} had its base case value (by design). The length of each electrode section $[2z_1,\ z_3-z_1$, and $z_w-z_3]$ was individually varied, while keeping the lengths of the other electrode sections unchanged and also keeping z_2 halfway between z_1 and z_3. The results indicate that any increase in Z_1, Z_3-Z_1 or Z_w-Z_3 causes less than a 1% change in $\Delta\psi_{m-}$, while a decrease in Z_1 causes a decrease in $\Delta\psi_{m-}$ that can be larger than 1%. The effect of decreasing z_3-z_1 is not reported because decreasing z_3-z_1 could cause the locations of each potential energy minima to no longer be suitably approximated as being at $z=\pm\frac{1}{2}(z_1+z_3)$. The effect of decreasing z_w-z_3 is also not reported because the effect may not be the same for a configuration that does not have vertical electrode walls capping each end electrode at $z=\pm z_w$. The values of r_{w0} and z_1 were simultaneously changed in proportion with each other such that the base case value for z_1/r_{w0} remained the same, while keeping z_3-z_1, z_w-z_3, and $z_2=\frac{1}{2}(z_1+z_3)$ unchanged. The results indicate that for Z_1/R_{w0} kept at its base case value of 2, $\Delta\psi_{m-}$ increases with R_{w0}/R_{w2} with an approximate dependence (fit to within a few percent) given by $\Delta\psi_{m-}\approx 0.5(R_{w0}/R_{w2})^{-0.15}\ln(R_{w0}/R_{w2})$ for $1<R_{w0}/R_{w2}\lesssim 10$. For example, the value of $\Delta\psi_{m-}$ equals 0.82 for $R_{w0}=10R_{w2}$.

Suppose an electron plasma produces a three-dimensional electric potential well as a result of being trapped within an electrode configuration similar to that shown in Fig. 1. The possibility of forming a partially non-neutral plasma by confining positive ions within the well is now explored by assuming that the ions would follow the Boltzmann density relation in all three dimensions within the well. The ions may follow the Boltzmann density relation in all three dimensions if, for example, the effect of the magnetic field on the ions is negligible as a result of the ion cyclotron radius being larger than the ion plasma radius. Even if the ion cyclotron radius is smaller than the ion plasma radius, radial ion diffusion may cause the ion plasma to relax to a Boltzmann density distribution in three dimensions. When applied in three dimensions using cylindrical coordinates and assuming azimuthal symmetry, the Boltzmann density relation can be written as

$$n_{+}(r,z)=n_{s+}e^{-Z_{+}e[\phi(r,z)-\phi(0,0)]/T_{+}}\Theta_z, \tag{3}$$

where it is assumed that the ion velocity distribution is Maxwellian at (0,0). Here, $n_{+}(r,z)$ denotes ion density at (r,z), n_{s+} equals the ion density at (0,0), Z_{+} is the ion charge state, and T_{+} is the ion temperature. The term Θ_z is included to take into account the fact that ion confinement may not take place. The value of Θ_z is defined to be unity only within

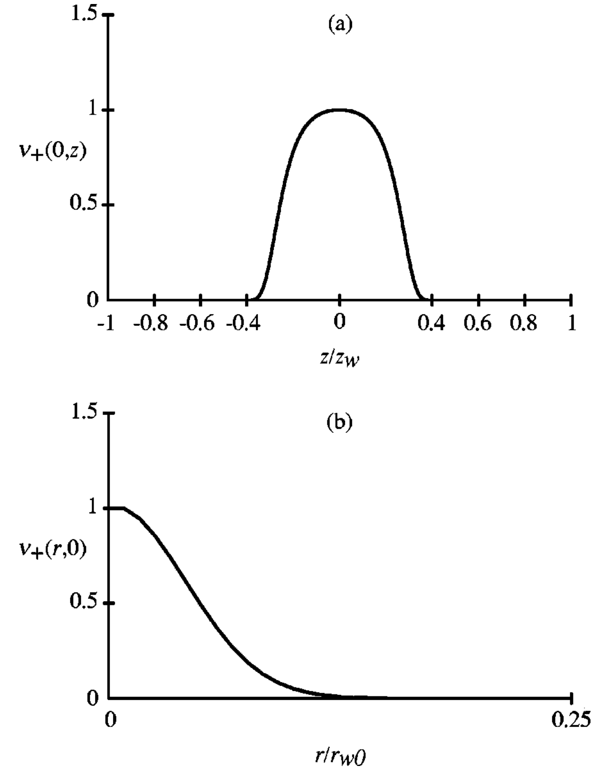

FIG. 4. Normalized ion density along the z axis (a) and along the midplane (b).

an electric potential well capable of confining ions and zero otherwise. A self-consistent equilibrium is computed as described above, except with $f(r,z)=(e/\epsilon_0)[n_{-}(r,z)-Z_{+}n_{+}(r,z)]$, where $n_{+}(r,z)$ is given by Eq. (3). By way of example, the following parameter values are selected for a "two-species base case:" $Z_{+}=1$, $n_{s+}=4.3\times10^{11}\ \mathrm{m}^{-3}$, and $T_{+}=300$ K. All of the other parameter values are the same as for the single-species base case. For the parameter values chosen, the electric potential and electron density are essentially the same as shown in Figs. 2 and 3. Plots of the normalized ion density $\nu_{+}(r,z)=n_{+}(r,z)/n_{s+}$ are shown in Fig. 4. The ion Debye length at (0,0) is equal to the electron Debye length at $(0,z_2)$. Also, the full width at half maximum for the axial ion density profile that is plotted in Fig. 4(a) is 30 times larger than the ion Debye length at (0,0), and the full width at half maximum for the radial ion density profile that is plotted in Fig. 4(b) is 10% larger than the ion Debye length at (0,0). In every direction away from the well's minimum at (0,0), the ion potential energy increases by an amount that is at least an order of magnitude larger than the ion temperature. With a potential energy well depth that is much larger than the ion temperature, the ion loss regions in velocity space are negligibly small at the bottom of the well, which is consistent with the assumption that the ions have an essentially full Maxwellian velocity distribution at (0,0).

The two-species equilibrium problem solved here can be stated using equations [i.e., Poisson's equation and Eqs. (1) and (3)] and boundary conditions that are expressed in terms of the dimensionless quantities ψ_{-}, R, Z, $\eta=n_{s-}/(Z_{+}n_{s+})$, $\tau=Z_{+}T_{-}/T_{+}$, R_p, R_{w0}, R_{w2}, Z_1, Z_2,

Z_3, Z_w, $\psi_-(R_{w0},0)$, and $\psi_-(R_{w2},Z_w)$. The parameter η defines the degree of partial neutralization at (0,0). In the limit of charge neutrality at (0,0), $\eta \to 1$, while a fully non-neutral plasma corresponds to $\eta \to \infty$. A solution of the two-species equilibrium problem in terms of the dimensionless quantities applies for any set of chosen parameter values provided the dimensionless parameters η, τR_p, R_{w0}, R_{w2}, Z_1, Z_2, Z_3, Z_w, $\psi_-(R_{w0},0)$, and $\psi_-(R_{w2},Z_w)$ remain unchanged. The results in Fig. 4 are in terms of $z/z_w = Z/Z_w$, $r/r_{w0} = R/R_{w0}$, and, by Eq. (3), $\nu_+ = e^{-\tau[\psi_-(R,Z)-\psi_-(0,0)]}$. Thus, the results in Fig. 4 also apply, for example, for the parameter values, $Z_+ = 19.23$, $n_{s+} = 2.2\times 10^{10}$ m^{-3}, and $T_+ = 0.5$ eV, if all of the other parameter values are the same as for the single-species base case. Note that the ion temperature can equal the electron temperature, $T_+ = T_- = 0.5$ eV, because Z_+ and n_{s+} are changed such that τ and η remain unchanged.

A parameter study was carried out aimed at determining the range of values for η and τ for which partially non-neutral plasma equilibria are self-consistently possible. To carry out the parameter study, a number of computations were done that were the same as the computation that produced the results for the two-species base case, except with a change in n_{s+} and/or T_+. The charge state Z_+ was not varied because the same effect on η and τ could be obtained by simultaneously changing n_{s+} and T_+. It was found that $\Delta\psi_{m-}$ increases with η. Thus, the fit expression for $\Delta\psi_{m-}$ obtained in the parameter study without the ions present provides the following condition with the ions present: $\Delta\psi_{m-} \lesssim 0.5(R_{w0}/R_{w2})^{-0.15} \ln(R_{w0}/R_{w2})$. At this point, it is helpful to define $\Delta\psi_{m+} = \tau\Delta\psi_{m-} = Z_+ e[\phi(r=0,z\approx z_2) - \phi(r=0,z=0)]/T_+$, which is the ratio of the axial depth of the ion potential energy well to the ion temperature. A necessary condition for ion loss regions in velocity space to be negligibly small at the bottom of the well is $\Delta\psi_{m+} \gg 1$. For this reason, partially non-neutral plasma equilibria are not considered possible if the condition $\Delta\psi_{m+} \gg 1$ does not hold. The conditions on $\Delta\psi_{m-}$ and $\Delta\psi_{m+}$ can be combined to yield the condition, $\tau^{-1} = T_+/(Z_+ T_-) \ll 0.5(R_{w0}/R_{w2})^{-0.15} \ln(R_{w0}/R_{w2}) \lesssim 1$. This is not a general condition, but one specific to the parameter study carried out (e.g., for $1 < R_{w0}/R_{w2} \lesssim 10$). Nevertheless, based on this condition, it appears likely that, for singly charged ions to be confined within a three-dimensional well produced by electrons, the ion temperature must be smaller than the electron temperature.

To determine the range of values for η for which partially non-neutral plasma equilibria are possible, various equilibria were obtained with $\tau \gg 1$. For each value of τ considered, it was found that η had an approximate minimum value for which the condition $\Delta\psi_{m+} \gg 1$ was valid. Although the results obtained are specific to the parameter study carried out, the results are consistent with the requirement that the ion charge density be less than the electron charge density for an ion-confining three-dimensional well to form.

Two specific equilibria are now considered, which are describable by the results shown in Figs. 2, 3, and 4. For the first equilibrium, the parameters are the same as chosen for the two-species base case, and the ions are considered to be singly charged xenon ions. If a 0.03 T magnetic field is assumed, then the ion cyclotron radius would be 26% larger than the radius of the electron plasma and thus larger than the radius of the ion plasma. In contrast, the electron cyclotron radius would be about two orders of magnitude smaller than the radius of the electron plasma. Consequently, it is reasonable to expect the ion density to follow Eq. (3) and the electron density to follow Eq. (1), at least in the limit that the interaction between the two species can be neglected. However, the effects associated with the interaction of the two oppositely signed plasma species may not be negligible, and such effects could be studied. One effect would be that the ion and electron temperatures would tend to equilibrate as a result of collisions between ions and electrons. Suppose the electrons are externally heated and fueled such that the electron temperature and density remain constant. Then, using the plasma parameters at (0,0) and Eqs. (15) and (16) of Ref. [3], the two-temperature equilibration time scale is calculated to be 5 min. For the calculation, the maximum impact parameter for a binary collision is set equal to the electron cyclotron radius (which is smaller than the ion cyclotron radius, the electron Debye length, and the ion Debye length), and the center-of-mass energy for a binary collision is approximated as being equal to the electron temperature. The calculated time scale is very large because of the effect that disparate masses have on the two-temperature equilibration time scale. If the singly charged xenon ions are replaced by positrons of the same density and temperature, then the two-temperature equilibration time scale is calculated to be 1 ms. For calculating the two-temperature equilibration time scale using positrons instead of xenon ions, the maximum impact parameter is set equal to the positron cyclotron radius, and the center-of-mass energy is set equal to twice the electron temperature. It is also interesting to consider that it may be possible to keep the temperatures of the two species from equilibrating. For example, if the electrons are externally heated, a cold buffer gas could be introduced that cools the ions by collisions.

So far, a positive plasma species has been considered to be trapped within a three-dimensional electric potential well that is produced by a negative plasma species. A specific equilibrium in which the roles of the positive and negative species are reversed is now considered. Suppose a singly charged xenon plasma is trapped with the parameters chosen for the single-species base case, except with a change of the sign of V_4. For an equilibrium comparable to the two-species base case, an electron plasma would be trapped within the three-dimensional well with a density and temperature of 4.3×10^{11} m^{-3} and 300 K, respectively, at (0,0). Assume that a 5 T magnetic field is present, that the ions are externally heated and fueled such that the ion temperature and density remain constant, and that the electrode structure that surrounds the partially non-neutral plasma is at room temperature. For the parameters considered, the two-temperature equilibration time scale is calculated to be about 30 s. For the calculation, the maximum impact parameter is set equal to the electron cyclotron radius and the center-of-mass energy is set equal to the electron temperature. The two-temperature equilibration time scale is more than two orders of magnitude larger than the electron cyclotron radiation time scale. Thus, the electron temperature should not equilibrate with the ion temperature. Instead, the electron temperature should remain near that of the surrounding electrode structure.

In summary, two types of plasma equilibria have been predicted for a variable-electrode-radius Malmberg-Penning trap with an electrode configuration similar to that illustrated in Fig. 1. The first type was for confinement of a single-species, fully non-neutral plasma. It was found that such a plasma could self-consistently produce a three-dimensional electric potential well, at least if the diameter of the plasma is only a few times larger than the Debye length associated with the plasma. The second type was for confinement of a two species, partially non-neutral plasma. One of the plasma species would be confined within a three-dimensional well produced by the other species. Various parameter studies were done and indicate that, if each plasma species is comprised of singly charged particles and $1 < R_{w0}/R_{w2} \lesssim 10$, then the species confined within the three-dimensional well must have a density and a temperature that are each smaller than the density and temperature, respectively, of the other plasma species. Two specific partially non-neutral plasma equilibria were considered, one with singly charged xenon ions confined within a three-dimensional well produced by electrons and the other with electrons confined within a three-dimensional well produced by singly charged xenon ions.

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